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Short Communication

A note on the acoustic character of time-variant one-port sources

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1. Introduction

One-port acoustical source in a duct system can be characterized by the acoustical source impedance and strength, usually using the linear frequency-domain modeling [1,2]. It is known that the direct method employing an array sensor layout yields different results from the indirect method, i.e., *load method*, in measuring the source parameters of various fluid machines. Although the real part of the source impedance is expected to be non-negative in physical sense, the indirect method sometimes comes up with negative source resistance values at some frequencies [3,4]. It is reported that the time-varying nature of the source is the major cause of a negative resistance [4–6] although the nonlinearity also contributes to the phenomenon to some extent [7].

In this paper, the acoustical characteristics of fluid machines are studied analytically, in which the sources are considered to be time-variant. For this purpose, a simple fluid machine consisting of a cylinder with a reciprocating piston and an exhaust (or, in general, connected) pipe system is taken as a simplified model representing a typical periodic, time-varying duct system. In addition, the equivalent circuit corresponding to such a simple model is analyzed.

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2. Theory

Fig. 1 shows the duct model for studying the characteristics of a time-varying source. The piston is operating in a reciprocating motion, which is similar to the compressor that was used for investigating the characteristics of measurement technique applicable to the time-varying source parameters in the previous work [8]. This simplified compressor is assumed to have a large degree of time-variance and also assumed to have no complicating factor such as mean flow or high temperature and its gradient. For simplicity in analysis, it is additionally assumed that the flow condition at the orifice is linear as well as the relationship between gas pressure and cylinder volume. The whole exhaust system except the moving piston is assumed linear and time-invariant. The source-load interface is positioned at a point just after the orifice in the downstream pipe although such a position may be determined elsewhere. If the plane wave propagation is assumed, the acoustic properties are constant in the duct section.

The equivalent electro-acoustic circuit for this source-load system is as represented in Fig. 2. Here, the mass inside the cylinder, $q(t)$, is given by

$$q(t) = \rho(t)V(t), \tag{1}$$

where $\rho(t)$ and $V(t)$ are the mass density and the volume in the cylinder, respectively. Mass flow from the cylinder to the connecting pipe, $u(t)$, is related to $q(t)$ as follows:

$$q(t) = -\int u(t)\,\mathrm{d}t. \tag{2}$$

The linear relationship between the mass density $\rho(t)$ and the pressure $p(t)$ in the cylinder can be expressed by the Hooke's law [9] as

$$p(t) - p_0 = B_{\mathrm{ad}}\left(\frac{\rho(t) - \rho_0}{\rho_0}\right), \tag{3}$$

or

$$\rho(t) = \frac{1}{c_0^2}\left(p(t) - p_0 + \rho_0 c_0^2\right). \tag{4}$$

Here, B_{ad} denotes the adiabatic bulk modulus, c_0 is the speed of sound, i.e., $c_0 = \sqrt{(B_{\mathrm{ad}}/.\rho_0)}$, and p_0 and ρ_0 are the steady-state components of pressure and density, respectively. The flow past the

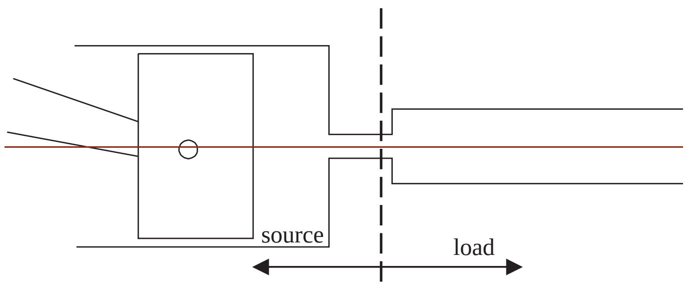

Fig. 1. Simplified compressor having a reciprocating piston and an exhaust pipe.

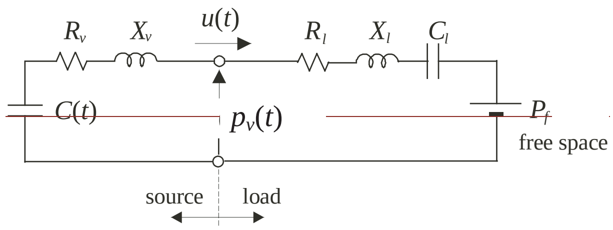

Fig. 2. Equivalent circuit for the source-load system.

orifice can be written as

$$p(t) - p_v(t) = R_v u(t) + X_v \frac{\mathrm{d}u(t)}{\mathrm{d}t}, \tag{5}$$

where R_v, X_v are the resistance and reactance terms of the orifice and $p_v(t)$ denotes the pressure at the exit of the orifice [10].

Assembling Eqs. (1)–(5), the following differential equation can be easily derived:

$$X_v \frac{\mathrm{d}^2 q(t)}{\mathrm{d}t^2} + R_v \frac{\mathrm{d}q(t)}{\mathrm{d}t} + \frac{c^2}{V(t)} q(t) = p_v(t) + \rho_0 c_0^2 - p_0. \tag{6}$$

When the exhaust system is being considered as an acoustic load, it can be regarded as a linear time-invariant system, for which the governing equation can be expressed as

$$X_l \frac{\mathrm{d}^2 q(t)}{\mathrm{d}t^2} + R_l \frac{\mathrm{d}q(t)}{\mathrm{d}t} + \frac{1}{C_l} q(t) = -p_v(t) + p_f, \tag{7}$$

where X_l, R_l, and C_l denote the effective inertance, resistance, and compliance, respectively, and p_f is the free-field pressure. Combining Eqs. (6) and (7), one can obtain a linear time-variant equation as

$$\frac{\mathrm{d}^2 q(t)}{\mathrm{d}t^2} + 2\zeta \frac{\mathrm{d}q(t)}{\mathrm{d}t} + [s_0 - s_1 g(t)] q(t) = F. \tag{8}$$

In Eq. (8), the following definitions of variables are used:

$$2\zeta \equiv \frac{R_v + R_l}{X_v + X_l}, \quad s_0 - s_1 g(t) \equiv \frac{1}{X_v + X_l}\left[\frac{c_0^2}{V(t)} + \frac{1}{C_l}\right], \tag{9a,b}$$

$$F \equiv \frac{1}{X_v + X_l}\left(p_f + \rho_0 c_0^2 - p_0\right). \tag{9c}$$

In Eq. (9b), s_0 implies the magnitude of steady component and s_1 the magnitude of time-varying one. The coefficient $g(t)$ is periodically varying with a period of $2\pi/\omega_p$. It is noted that Eq. (8) is actually the second-order Hill's equation with a constant forcing term. Although the solution is well known for the cases of simple functions of $g(t)$ [11], the closed form solutions for general $g(t)$ are difficult to find. However, it is still possible to obtain some information on the source character by inspecting the stability condition of the solutions.

By using the transformation of $x(t) = \exp(\zeta t)q(t)$ and $\omega_p t = 2\xi$, the homogeneous form of Eq. (8) can be converted into the following expression that do not contain any term with a first derivative:

$$x''(\xi) + [\alpha - 2\beta g(\xi)]x(\xi) = 0. \tag{10}$$

Here, $g(\xi)$ is periodic in ξ with a period of π and the following definitions of α and β are used:

$$\alpha = \frac{4}{\omega_p^2}\left(s_0 - \varsigma^2\right), \quad \beta = \frac{2}{\omega_p^2}s_1. \tag{11a,b}$$

The stability condition can be found by investigating the properties of the characteristic exponents or eigenvalues of the discrete transition matrix [11]. For the second order, periodically time-varying systems, in the case of a small time-varying component, the following condition for instability and gain would hold in general:

$$\alpha = n^2 \quad (n = 1, 2, 3, \ldots). \tag{12}$$

If the damping effects are neglected (i.e., $\zeta \approx 0$), Eq. (12) can be rewritten as

$$f_p \equiv \frac{\omega_p}{2\pi} = \frac{1}{2\pi}\left(\frac{2\sqrt{s_0}}{n}\right) = \frac{2f_{\mathrm{res}}}{n}, \tag{13}$$

where f_{res} is equal to $\sqrt{s_0}/2\pi$, namely, the resonance frequency of the 'static' source-load system. The 'static' system implies that the piston stands motionless at the midpoint of its stroke. When the time-varying component is small, the source radiates a large acoustic power if the piston operates at twice the natural frequency of the static system or integral submultiples of that rate. Therefore, designers for the silencing system should try to avoid the condition of Eq. (13) in the operating ranges of the engine speed by modifying the source or load impedances. If the damping effects are not small, the condition of large radiation power can be found by using the stability diagrams [11].

It is worthwhile comparing the foregoing results with those of the conventional linear time-invariant source model. Using Eq. (7), the load impedance $Z_l(f)$ can be written as

$$Z_l(f) = \frac{P_v(f)}{u(f)} = R_l + \mathrm{i}\left[2\pi f X_l - \frac{1}{2\pi f C_l}\right], \tag{14}$$

where i is $\sqrt{-1}$ and f is the frequency. The source impedance $Z_s(f)$ can be obtained by using Eq. (6) and neglecting the time-varying component of $V(t)/c_0^2$ as

$$Z_s(f) = R_v + \mathrm{i}\left[2\pi f X_v - \frac{1}{2\pi f C_0}\right], \tag{15}$$

where C_0 denotes the steady component of $V(t)/c_0^2$. It is well known that the linear time-invariant source radiates the maximum acoustic power when the load impedance equals the conjugate of the source impedance [12], that is,

$$Z_l(f) = Z_s^*(f). \tag{16}$$

Eq. (16) can be reexpressed by using Eqs. (14) and (15) for small resistance values as

$$f = \frac{1}{2\pi}\sqrt{\frac{C_0 + C_l}{C_0 C_l (X_v + X_l)}} = \frac{\sqrt{s_0}}{2\pi} = f_{\rm res}. \tag{17}$$

Eq. (17) can be also obtained by inspecting the natural frequency of the system in Eq. (10) provided that the time-varying component β is zero. A maximum acoustic power is radiated from the source when the source operates at the resonance frequency of the static source-load system. It is noted that the condition in Eq. (17) is identical to Eq. (13).

3. Conclusions

In order to study the acoustical characteristics of in-duct time-variant sources, a simplified fluid machine comprised of a reciprocating piston and an exhaust is considered and the equivalent electro-acoustic circuit is analyzed. The pressure response when a load is applied to the source can be obtained by solving the second-order Hill's equation with a constant forcing term. It is found that, for a small magnitude of the time-varying component, the source radiates a large acoustic power if the piston operates at twice the natural frequency of the static system, or integral submultiples of that rate. The results can be useful in refining the acoustical source models and developing the low-noise intake or exhaust system of fluid machines.

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